#### Inclusive Heavy-Flavor Production from Nuclei

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#### Abstract

We describe a light-cone wave function formulation for hadroproduction of heavy-flavors at high energies. At moderate values of  $x_F$  heavy-flavor production can be viewed as a diffractive excitation of heavy quark-antiquark Fock states, present in the interacting gluon from the projectile. The approach developed here is well suited to address coherence effects in heavy-quark production from nuclei at small values of  $x_t \lesssim 0.1 \cdot A^{-1/3}$ .

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#### 1 Introduction

The inclusive production of heavy-flavor quark-antiquark  $(Q\bar{Q})$  pairs in highenergy hadron-hadron collisions is one of the standard applications of perturbative QCD. Calculations, available in next to leading order in the strong coupling constant  $\alpha_s$ , agree reasonably well with experimental data (for a recent review see ref.[1]). In the perturbative treatment of heavy-flavor production from nuclear targets, nuclear modifications of the total production cross section are usually neglected. Although this is certainly sufficient on a qualitative level, it is hard to justify quantitatively within the framework of perturbative QCD, especially since it demands the calculation of higher order Feynman diagrams which involve different nucleons inside the nucleus. However the light-cone wave function formalism outlined in this paper is ideal for an investigation of nuclear effects. In this formulation heavy-quark production at large energies resembles diffractive processes. More insight is gained about the propagation of QQ pairs through the nuclear medium. This matter is closely related to the effects of "color transparency" and "color opacity", which are discussed in a large variety of processes, ranging from quasielastic electron scattering to heavy quarkonium production (see e.g. [2, 3] and references therein).

Let s be the squared center of mass energy of the collision process. The Feynman variable  $x_F$  is defined as  $x_F = 2P_{Q\bar{Q}}/\sqrt{s}$ , where  $P_{Q\bar{Q}}$  is the longitudinal momentum of the QQ pair in the center of mass of the reaction. It is common to introduce the variables  $x_t$  and  $x_b$  via:

$$x_t x_b s = M_{Q\bar{Q}}^2,$$
 (1)  
 $x_F = x_b - x_t,$  (2)

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with  $M_{Q\bar{Q}}$  being the invariant mass of the produced  $Q\bar{Q}$ . In perturbative QCD  $x_b$  and  $x_t$  are identical to the light-cone momentum fractions of the active beam and target parton, respectively. Within the leading order perturbative scheme they enter in the heavy-quark production cross section as follows:

$$\sigma_{Q\bar{Q}}(s) = \sum_{i,j} \int dx_t dx_b \, f_t^i(x_t, \mu^2) \, f_b^j(x_b, \mu^2) \, \sigma_{ij}(x_t x_b s, \mu^2) \,. \tag{3}$$

Here  $f_t^i(x_t,\mu^2)$  and  $f_b^j(x_b,\mu^2)$  are the densities of partons "i" and "j", carrying fractions  $x_t$  and  $x_b$  of the light-cone momenta of the colliding target and projectile. The partonic subprocess  $i+j \to Q+\bar{Q}$  is described by the cross section  $\sigma_{ij}(x_tx_bs,\mu^2)$ . It requires a squared center of mass energy  $x_t x_b s = M_{Q\bar{Q}}^2 \ge 4m_Q^2$ , where  $m_Q$  is the invariant mass of the heavy quark. The factorization in parton densities and hard partonic subprocesses is carried out at a typical scale  $\mu^2 \sim 4 m_Q^2$ .

In this work we will be concerned with nuclear effects in inclusive hadroproduction of  $Q\bar{Q}$  pairs which carry a high energy in the laboratory frame. In particular we consider processes at small target light-cone momentum fractions  $x_t < 0.1$ . Furthermore we will restrict ourselves to moderate  $x_F$ . Thus we concentrate on the kinematic domain where  $Q\bar{Q}$  production is dominated by the gluon fusion subprocess  $g + g \to Q + \bar{Q}$  (see e.g. ref.[4]) and neither the annihilation of light-quarks nor the excitation of higher-twist intrinsic heavy-quark components [5] are of importance. The perturbative QCD cross section (3) is then proportional to the density of gluons in beam and target. In the case of open-charm production at typical Tevatron energies ( $s \sim 1600 \, GeV^2$ ) this implies that we focus on the region  $0 < x_F < 0.5$ .

The paper is organized as follows: In Sec. 2 we discuss the space-time picture of heavy-quark production at small  $x_t$ , as seen from the lab frame. Section 3 introduces the light-cone wave function formulation of heavy-flavor production from free nucleon targets. This approach is extended to nuclear targets in Sec. 4. Finally we summarize the main results in Sec. 5.

# 2 Lab frame picture of $Q\bar{Q}$ production at small $x_t$

At moderate values of  $x_F$  heavy-quark production proceeds via the fusion of a projectile and target gluon. In the lab frame, where the target is at rest, the projectile gluon interacts at small  $x_t < 0.1$  via quark-antiquark fluctuations present in its wave function. In heavy-flavor production only heavy  $Q\bar{Q}$  Fock states of the incident gluon are of relevance. Their propagation length is typically of the order:

$$l_{Q\bar{Q}} \sim \frac{2\nu_G}{4m_O^2} \approx \frac{1}{Mx_t},\tag{4}$$

where  $\nu_G = x_b E_{lab}$  denotes the lab frame energy of the projectile gluon and  $E_{lab}$  stands for the beam energy. For  $Q\bar{Q}$  production from a nuclear target with mass number A and radius  $R_A$  the propagation length  $l_{Q\bar{Q}}$  plays an important role. If  $l_{Q\bar{Q}}$  is less then the average nucleon-nucleon distance in nuclei,  $Q\bar{Q}$  production takes place incoherently on all nucleons inside the target. Consequently then nuclear effects in inclusive heavy-flavor production will be absent. However if the energy of the incident gluon is large, or equivalently  $x_t < 0.1 \cdot A^{-1/3}$  is small, the coherence length will exceed the nuclear size,  $l_{Q\bar{Q}} > R_A$ . Then the formation of  $Q\bar{Q}$  pairs takes place

coherently on the whole nucleus. This may in principle lead to nuclear modifications of the heavy-flavor total production cross section. <sup>1</sup> The magnitude of these nuclear effects however is controlled by the transverse size of the  $Q\bar{Q}$  fluctuations and turns out to be small as discussed below.

Once produced, a  $Q\bar{Q}$  pair evolves into hadrons after a typical formation length

$$l_f \sim \frac{2\pi}{\Delta M_{Q\bar{Q}}} \frac{\nu_G}{M_{Q\bar{Q}}} \gg l_{Q\bar{Q}}.$$
 (5)

Here  $\Delta M_{Q\bar{Q}}$  is the characteristic mass difference of heavy  $Q\bar{Q}$  states. Since the formation time  $l_f$  is much larger than the coherence length  $l_{Q\bar{Q}}$ , it is justified to neglect the intrinsic evolution of a produced  $Q\bar{Q}$  pair during its propagation through the nucleus.

At large  $x_F \sim 1$  the momentum of the projectile is transfered collectively to a heavy  $Q\bar{Q}$  state by several partons in the beam hadron [5]. In this case the heavy quark pair can evidently not be assigned to a single projectile gluon. Consequently, the excitation of heavy-quark components at  $x_F \sim 1$  is quite different from the  $Q\bar{Q}$  production mechanism at moderate  $x_F$ . In this work we focus on moderate  $x_F$  only, where the above mechanism is not important.

## 3 $Q\bar{Q}$ production from free nucleons

As a first step we consider the production of heavy quark-antiquark pairs through the interaction of projectile gluons with free nucleons. The total  $Q\bar{Q}$  production cross section for free nucleon targets is then obtained by a convolution with the incident gluon flux.

To leading order in the QCD coupling constant  $\alpha_s$ , the incoming projectile gluon has a simple Fock state decomposition. It includes bare gluons, two-gluon states and quark-antiquark components. At small  $x_t < 0.1$  the propagation lengths of these Fock states exceed the target size as pointed out in the previous section. Furthermore, at large projectile energies the transverse separations and the longitudinal momenta of all partons in a certain Fock component are conserved during its interaction with the target [6]. Consequently it is most convenient to describe the different Fock components in terms of light-cone wave functions in a "mixed" representation, given by these conserved quantities. In this representation the incident,

<sup>&</sup>lt;sup>1</sup>Note, that this picture is quite similar to deep-inelastic scattering at small values of the Bjorken variable x, leading to nuclear shadowing (see e.g. [6, 7]).

dressed gluon  $|G(k_h^2)\rangle$  can be decomposed as: <sup>2</sup>

$$|G(k_b^2)\rangle = \sqrt{1 - n_{Q\bar{Q}} - n_{\xi}}|g\rangle + \sum_{z,\vec{r}} \Psi_G(z,\vec{r})|Q\bar{Q};z,\vec{r}\rangle + \sum_{\xi} \Psi(\xi)|\xi\rangle. \quad (6)$$

Here  $\Psi_G(z,\vec{r})$  is the projection of the dressed gluon wave function onto a  $Q\bar{Q}$  state in the  $(z,\vec{r})$ -representation, where the light-cone variable z represents the fraction of the gluon momentum carried by the quark and  $\vec{r}$  is the transverse separation of the Q– $\bar{Q}$  in the impact parameter space. The light quark-antiquark  $(q\bar{q})$  and gluon-gluon (gg) Fock components are written as  $\sum_{\xi} \Psi(\xi) |\xi\rangle$ . Since they are of no importance for our considerations we will omit them from now on. The normalization of the  $Q\bar{Q}$  Fock states is given by:

$$n_{Q\bar{Q}} = \int_0^1 dz \int d^2 \vec{r} |\Psi_G(z, \vec{r})|^2.$$
 (7)

Similarly,  $n_{\xi}$  denotes the normalization of the light-flavor  $(q\bar{q} \text{ and } gg)$  components. The  $Q\bar{Q}$  wave function  $\Psi_G$  of the gluon can be obtained from the  $Q\bar{Q}$  wave function of a photon  $\Psi_{\gamma^*}$ , as derived in ref.[6]. The only difference is a color factor and the substitution of the strong coupling constant for the electromagnetic one:

$$|\Psi_G(z, \vec{r})|^2 = \frac{\alpha_s(r)}{6 \alpha_{em}} |\Psi_{\gamma^*}(z, \vec{r})|^2$$

$$= \frac{\alpha_s(r)}{(2\pi)^2} \left\{ [z^2 + (1-z)^2] \epsilon^2 K_1^2(\epsilon r) + m_Q^2 K_0^2(\epsilon r) \right\}. \quad (8)$$

 $K_{0,1}$  are the modified Bessel functions,  $\alpha_s(r)$  is the running coupling constant in coordinate-space,  $r = |\vec{r}|$ , and  $\epsilon^2 = z(1-z)k_b^2 + m_Q^2$ .

The scattering of an incident gluon carrying color a from a target nucleon at an impact parameter  $\vec{b}$  is described by the scattering matrix  $\hat{S}(\vec{b})$ :

$$|G^a, N; \vec{b}\rangle \Longrightarrow \hat{S}(\vec{b})|G^a, N; \vec{b}\rangle.$$
 (9)

In the impact parameter representation we use the S-matrix in the eikonal form from ref.[8]:

$$\hat{S}(\vec{b}) = \exp\left[-i\sum_{i,j} V(\vec{b} + \vec{b}_i - \vec{b}_j)\hat{T}_i^b \hat{T}_j^b\right],$$
(10)

with the one-gluon exchange potential (eikonal function)

$$V(\vec{b}) = \frac{1}{\pi} \int d^2 \vec{k} \, \frac{\alpha_s(\vec{k}^2) \, e^{i\vec{k}\cdot\vec{b}}}{\vec{k}^2 + \mu_q^2}.$$
 (11)

<sup>&</sup>lt;sup>2</sup> Wherever it does not lead to confusion we suppress the virtuality of the incident gluon  $k_b^2$ .

 $\hat{T}_{i,j}^b$  are color SU(3) generators acting on the individual partons of the projectile and target at transverse coordinates  $\vec{b}_i$  and  $\vec{b}_j$  respectively. The effective gluon mass  $\mu_g$  is introduced as an infrared regulator. In heavy-flavor production  $m_Q\gg\mu_g$ , which ensures that our final results will not depend on the exact choice of  $\mu_g$ . Furthermore, as we will show below,  $\mu_g$  can be absorbed into the definition of the target gluon density. In Eq.(10) and below an implicit summation over repeated color indices is understood. In the lowest non-trivial order the scattering matrix in (10) accounts for the exchange of two t-channel gluons between the target and projectile. This has turned out to be successful in describing high energy hadron-hadron forward scattering processes [8], deep-inelastic scattering at small values of the Bjorken variable x [6], and the diffractive photoproduction of vector mesons [9]. In Eq.(9) the leading term is:

$$\begin{split} \left(\hat{S}(\vec{b}\,) - 1\right) |G^a, N; \vec{b}\,\rangle = \\ \frac{-i}{\pi} \int d^2\vec{k} \, \frac{\alpha_s(\vec{k}^{\,\,2}) \, e^{i\vec{k}\cdot\vec{b}}}{\vec{k}^2 + \mu_g^2} \left[ \sum_j e^{-i\vec{k}\cdot\vec{b}_j} \hat{T}^b_j \, |N\rangle \right] \left[ \sum_i e^{i\vec{k}\cdot\vec{b}_i} \hat{T}^b_i \, |G^a\rangle \right] (12) \end{split}$$

The last term in Eq.(12) represents the coupling of an exchanged t-channel gluon with color b to the incident gluon, while the second to last term describes the coupling to the nucleon target. The interaction of the incident gluon can be expressed in terms of its Fock components specified in Eq.(6). For this purpose, note that the quark and antiquark of a certain  $Q\bar{Q}$  Fock state with transverse distance  $\vec{r}$  and light-cone momentum fraction z are located at impact parameters  $\vec{b}_Q = \vec{r}(1-z)$  and  $\vec{b}_{\bar{Q}} = -\vec{r}z$  with respect to the parent gluon. Neglecting light quark-antiquark and gluon-gluon components we obtain (see Fig. 1):

$$\sum_{i} e^{i\vec{k}\cdot\vec{b}_{i}} \hat{T}_{i}^{b} |G^{a}\rangle = if_{abc}\sqrt{1 - n_{Q\bar{Q}}} |g^{c}\rangle 
+ \sum_{\vec{r},z} \Psi_{G}(z,\vec{r}) \left\{ \frac{1}{2} \left( e^{i\vec{k}\cdot\vec{r}(1-z)} + e^{-i\vec{k}\cdot\vec{r}z} \right) if_{abc} \left| Q\bar{Q}_{[8]}^{c} \right\rangle \right. 
\left. + \frac{1}{2} \left( e^{i\vec{k}\cdot\vec{r}(1-z)} - e^{-i\vec{k}\cdot\vec{r}z} \right) d_{abc} \left| Q\bar{Q}_{[8]}^{c} \right\rangle 
+ \frac{1}{\sqrt{6}} \left( e^{i\vec{k}\cdot\vec{r}(1-z)} - e^{-i\vec{k}\cdot\vec{r}z} \right) \delta_{ab} \left| Q\bar{Q}_{[1]} \right\rangle \right\}.$$
(13)

As usual  $f_{abc}$  and  $d_{abc}$  are the antisymmetric and symmetric SU(3) structure constants. In terms of the color wave functions of its quark and antiquark constituents the color wave function of an octet  $Q\bar{Q}$  pair with color c is:

$$\left| Q \bar{Q}_{[8]}^c \right\rangle = \sqrt{2} \sum_{k,l} \hat{T}_{kl}^c \left| Q_k \right\rangle \left| \bar{Q}_l \right\rangle. \tag{14}$$

The indices k and l specify the color of the quark and antiquark. Similarly, a color singlet  $Q\bar{Q}$  state is given by:

$$\left| Q\bar{Q}_{[1]} \right\rangle = \frac{1}{\sqrt{3}} \sum_{k} \left| Q_k \right\rangle \left| \bar{Q}_k \right\rangle.$$
 (15)

The scattering state (13) can be decomposed into final state, dressed gluons and heavy-quark pairs:

$$\sum_{i} e^{i\vec{k}\cdot\vec{b}_{i}} \hat{T}_{i}^{b} |G^{a}\rangle = if_{abc} |G^{c}\rangle 
+ \sum_{\vec{r},z} \Psi_{G}(z,\vec{r}) \left\{ \frac{1}{2} \left( e^{i\vec{k}\cdot\vec{r}(1-z)} + e^{-i\vec{k}\cdot\vec{r}z} - 2 \right) if_{abc} |Q\bar{Q}_{[8]}^{c}\rangle \right. 
+ \frac{1}{2} \left( e^{i\vec{k}\cdot\vec{r}(1-z)} - e^{-i\vec{k}\cdot\vec{r}z} \right) d_{abc} |Q\bar{Q}_{[8]}^{c}\rangle 
+ \frac{1}{\sqrt{6}} \left( e^{i\vec{k}\cdot\vec{r}(1-z)} - e^{-i\vec{k}\cdot\vec{r}z} \right) \delta_{ab} |Q\bar{Q}_{[1]}\rangle \right\}.$$
(16)

In this decomposition the last three terms represent heavy-quark pairs produced via the interaction of the incident gluon with the exchanged t-channel gluon. According to the derivation leading to Eq.(16) they have to be seen as diffractive excitations of heavy-flavor  $Q\bar{Q}$  Fock components present in the wave function of the incident dressed gluon. The contribution of these  $Q\bar{Q}$  Fock states to the heavy-flavor production amplitude disappears if their transverse size  $\vec{r}$  vanishes, since then they cannot be resolved by the exchanged t-channel gluon. This is due to the fact that interactions of point-like color octet  $Q\bar{Q}$  states are indistinguishable from interactions of color octet gluons, as a consequence of color gauge invariance. Each of the color octet  $Q\bar{Q}$  terms proportional to f and d, as well as the color singlet term in Eq.(16), vanishes for  $\vec{r} \to 0$ .

There is a one-to-one correspondence between the configuration-space derivation of heavy-flavor production, and the conventional perturbative QCD approach (see Fig. 1). The excitation of color singlet  $Q\bar{Q}$  states receives contributions only from t- and u-channel quark exchange diagrams (Fig. 1a and Fig. 1b). The same is true for the excitation of color octet  $Q\bar{Q}$  pairs proportional to the color factor d. The excitation of color octet  $Q\bar{Q}$  states proportional to f receives contributions also from the s-channel gluon exchange process in Fig. 1c. They yield the term "-2" within the factor  $(e^{i\vec{k}\cdot\vec{r}(1-z)}+e^{-i\vec{k}\cdot\vec{r}z}-2)$  in Eq.(16). This s-channel gluon-exchange contribution is crucial for the disappearance of the corresponding heavy-quark excitation amplitude at  $\vec{r} \to 0$ .

In the following we focus on the diffractively produced  $Q\bar{Q}$  states and omit the gluon contribution in Eq.(16). In the light of our discussion in

Sec. 2 this should yield the main contribution to heavy-flavor production at high energies and moderate  $x_F$ . In the lowest non-trivial order we obtain the heavy-flavor production cross section in gluon-nucleon collisions by inserting the  $Q\bar{Q}$  component of Eq.(16) into Eq.(12), multiplying with the complex conjugate, integrating over the impact parameter space, summing over the color of the final state and averaging over the color of the incident gluon:

$$\sigma(GN \to Q\bar{Q}X) = \frac{2}{3} \int d^2\vec{r} \int_0^1 dz \, |\Psi_G(z, \vec{r})|^2 \int d^2\vec{k} \, \frac{\alpha_s^2(\vec{k}^2) \, \mathcal{F}(k^2)}{(k^2 + \mu_g)^2} \times \left(17 - 9e^{-i\vec{k}\cdot\vec{r}z} - 9e^{i\vec{k}\cdot\vec{r}(1-z)} + e^{i\vec{k}\cdot\vec{r}}\right). \tag{17}$$

All relevant information about the target is in  $\mathcal{F}(k^2)$ , which is linked to the form factor  $G_2$  related to the coupling of two gluons to the nucleon:

$$\mathcal{F}(k^2) = 1 - G_2(k, -k) = 1 - \langle N | e^{i\vec{k} \cdot (\vec{b}_1 - \vec{b}_2)} | N \rangle.$$
 (18)

The vectors  $\vec{b}_1$  and  $\vec{b}_2$  specify the coordinates of the interacting quarks in the impact parameter space. If the size of the target, which carries no net color, would shrink to zero, the exchanged gluons would decouple from the target and  $\mathcal{F} = 0$ . In actual calculations we determine  $G_2$  using a constituent quark wave function for the nucleon of Gaussian shape, fitted to the electromagnetic charge radius of the nucleon.

The result in Eq.(17) can be illustrated as follows: Consider a color singlet gluon-quark-antiquark  $(gQ\bar{Q})$  state with a Q-g and  $\bar{Q}$ -g separation  $\vec{\rho}$  and  $\vec{R}$  respectively. The Q- $\bar{Q}$  separation is then  $\vec{r} = \vec{\rho} - \vec{R}$ . Furthermore let  $\sigma(r)$  be the cross section for the interaction of a  $Q\bar{Q}$  color dipole of size r with the nucleon target. In the Born approximation, i.e. with two gluon exchange, one has [8, 6]:

$$\sigma(r) = \frac{16}{3} \int d^2 \vec{k} \, \frac{\alpha_s^2(\vec{k}^2) \, \mathcal{F}(k^2)}{(k^2 + \mu_q^2)^2} \left( 1 - e^{i\vec{k}\cdot\vec{r}} \right). \tag{19}$$

Then the  $gQ\bar{Q}$ -nucleon cross section can be written as [10]

$$\sigma_{gQ\bar{Q}}^{N}(r,R,\rho) = \frac{9}{8} \left[ \sigma(R) + \sigma(\rho) \right] - \frac{1}{8} \sigma(r). \tag{20}$$

Comparing with Eq. (17) we find:

$$\sigma(GN \to Q\bar{Q}X) = \int d^2\vec{r} \int_0^1 dz \, |\Psi_G(z, \vec{r})|^2 \sigma_{gQ\bar{Q}}^N(r, -zr, (1-z)r) \,. \tag{21}$$

Another derivation of this result, based on unitarity, can be found in ref.[11].

Let us briefly discuss the properties of  $\sigma(GN \to Q\bar{Q}X)$ . In this respect it is useful to observe that Eq.(21) resembles heavy-flavor contributions to real and virtual photoproduction [6, 10]. The wave functions corresponding to these processes are equal, up to normalization factors (see Eq.(8)). While the three-parton cross section (20) enters in hadroproduction, the dipole cross section (19) is present in photoproduction. Both cross sections are closely related through Eq.(20). In the leading  $\log Q^2$  approximation the dipole cross section (19) is proportional to the gluon distribution of the target [10, 12, 13]:

$$\sigma(r) \to \sigma(x_t, r) = \frac{\pi^2}{3} r^2 \alpha_s(r) \left[ x_t g_t(x_t, k_t^2 \sim \frac{1}{r^2}) \right]. \tag{22}$$

The explicit dependence on  $x_t$  results from higher order Fock components of the projectile parton, e.g.  $Q\bar{Q}g$  states. Note that the gluon distribution in Eq.(22) absorbs the infrared regularization  $\mu_g$ . Most important for our further discussion is the color transparency property of the dipole cross section, i.e. its proportionality to  $r^2$ .

From the asymptotic properties of the modified Bessel functions one finds immediately that the squared  $Q\bar{Q}$  wave function decreases exponentially for  $r>1/\epsilon$ . We therefore conclude that for  $k_b^2\lesssim 4m_Q^2$  small transverse sizes are relevant for the  $Q\bar{Q}$  production process:

$$r^2 \lesssim \frac{1}{m_Q^2} \,. \tag{23}$$

For highly virtual gluons,  $k_b^2 \gg 4m_Q^2$ , the  $Q\bar{Q}$  wave function selects quark pairs with a transverse size  $r^2 \ll 1/4m_Q^2$ . Their contribution to the production cross section vanishes like  $\sim 1/k_b^2$  due to the color transparency property of the dipole cross section in Eq.(22). Combining Eqs.(21,22, 23) we find:

$$\sigma(GN \to Q\bar{Q}X) \propto x_t g_t(x_t, \mu^2 \sim 4m_Q^2)$$
. (24)

Equation (24) will later be important for a comparison with the parton model description of heavy-flavor production in Eq.(3).

An important property of  $\sigma(GN \to Q\bar{Q}\,X)$  is its infrared stability, i.e. its convergence in the limit  $\vec{k} \to 0$ . This is due to the fact that soft t-channel gluons with  $\vec{k} \to 0$  cannot resolve the  $Q\bar{Q}$  component of the incident beam gluon, and therefore cannot contribute to heavy-flavor production.

Although  $n_{Q\bar{Q}}$ , the normalization of a dressed gluon to be found in a  $Q\bar{Q}$  Fock state (7), diverges logarithmically at  $r\to 0$ , the cross section  $\sigma(GN\to Q\bar{Q}\,X)$  is finite. This is again implied by color transparency, since small size  $Q\bar{Q}$  pairs cannot be resolved by interacting t-channel gluons with

wavelengths  $\lambda \gtrsim r$ . They are therefore indistinguishable from bare gluons and cannot be excited into final  $Q\bar{Q}$  states.

We are now in the position to write down the heavy-quark production cross section  $\sigma_{Q\bar{Q}}(h,N)$  for hadron-nucleon collisions. For this purpose we have to multiply  $\sigma(GN \to Q\bar{Q}X)$  from Eq. (21) by the gluon density of the incoming hadron projectile and integrate over the virtuality  $k_b^2$  of the projectile gluon:

$$\frac{d\sigma_{Q\bar{Q}}(h,N)}{dx_b} = \int \frac{dk_b^2}{k_b^2} \frac{\partial [g_b(x_b,k_b^2)]}{\partial \log k_b^2} \, \sigma(GN \to Q\bar{Q}\,X; x_t, k_b^2). \tag{25}$$

As mentioned above, the leading contributions to  $\sigma(GN \to Q\bar{Q}X)$  result from the region  $k_b^2 \lesssim 4m_Q^2$ . With the approximation  $\sigma(GN \to Q\bar{Q}X; x_t, k_b^2) \approx \sigma(GN \to Q\bar{Q}X; x_t, k_b^2 = 0)$  we obtain:

$$\frac{d\sigma_{Q\bar{Q}}(h,N)}{dx_b} \approx g_b(x_b, \mu^2 = 4m_Q^2) \, \sigma(GN \to Q\bar{Q} \, X; x_t, k_b^2 = 0) \,. \tag{26}$$

Equations (24,26) describe the dependence of the production cross section on the beam and target gluon densities, in close correspondence to the conventional parton model ansatz of Eq.(3). This demonstrates the similarity between the light-cone wave function formulation and the conventional parton model approach. Since the  $g + g \rightarrow Q + \bar{Q}$  cross section decreases rapidly with the invariant mass of the heavy-quark pair, one finds dominant contributions to heavy-flavor production for  $x_t x_b s \approx 4m_O^2$ . This leads to:

$$\frac{d\sigma_{Q\bar{Q}}(h,N)}{dx_F} \approx \left(1 + \frac{4m_Q^2}{sx_b^2}\right) g_b(x_b, 4m_Q^2) \,\sigma(GN \to Q\bar{Q}\,X; x_b - x_F) \,, \quad (27)$$

with 
$$x_b \approx x_F/2 + \sqrt{16m_Q^2 s + s^2 x_F^2}/2s$$
.

Let us explore the result (27) for charm production in nucleon-nucleon collisions. To be specific we choose a beam energy  $E_{lab} = 800 \,\text{GeV}$ , as used in the Fermilab E743 experiment [14]. For the dipole cross section which enters in Eqs.(20,21) we employ two different parameterizations. First we use  $\sigma(r)$  from Eq.(19) with  $\mu_g = 0.140 \,\text{GeV}$ . This choice reproduces measured hadron-nucleon cross sections at high energies. Furthermore it has been successfully applied to nucleon structure functions at small x and moderate  $Q^2$ , and to nuclear shadowing [6].

As an alternative we use a parameterization from ref.[15] which includes the effects of higher order Fock states of the projectile parton. This choice has been shown to yield a good description of the small-x proton structure function measured at HERA. For transverse sizes  $r^2 \sim 1/m_c^2$  one finds in

the region  $x_t \lesssim x_0 = 0.03$  [15]:

$$\sigma(x_t, r) \approx \sigma(r) \left(\frac{x_0}{x_t}\right)^{\Delta},$$
 (28)

with  $\mu_g = 0.75 \,\text{GeV}$  and  $\Delta = 0.4$ . In both parameterizations the squared coupling constant  $\alpha_s^2(\vec{k}^2)$  appearing in Eq.(19) has been replaced by  $\alpha_s(r) \, \alpha_s(\vec{k}^2)$ .

The invariant mass of the charm quark is fixed at  $m_c=1.5\,\mathrm{GeV}$ . The gluon distribution of the nucleon projectile enters at moderate to large values of  $x_b$ . We therefore use the parameterization of ref.[16]. In Fig.2 we compare our results with the data from the E743 proton-proton experiment [14]. In the kinematic region  $0 < x_F < 0.5$ , where our approach is well founded, both parameterizations of the dipole cross section lead to a reasonable agreement with the experimental data.

# 4 Hadroproduction of $Q\bar{Q}$ pairs from nuclear targets

An important aspect of the light-cone wave function formulation of heavy-flavor production is the factorization of the  $Q\bar{Q}$  wave function and its interaction cross section (see Eq.(21)). This feature is crucial for the generalization to nuclear targets. The derivation of the  $Q\bar{Q}$  production cross section in (21) has been based upon the observation that the coherence length  $l_{Q\bar{Q}}$  of a  $Q\bar{Q}$  fluctuation, belonging to the incident gluon, is larger than the target size for high energies and small  $x_t$ . As a consequence the transverse size of the heavy-quark pair is frozen during the interaction process. This has led to a diagonalization of the scattering  $\hat{S}$ -matrix in a mixed  $(z,\vec{r})$ -representation. In high energy hadron-nucleus collisions with  $l_{Q\bar{Q}} \gtrsim R_A$ , this diagonalization of the S-matrix is also possible. Consequently we obtain the  $Q\bar{Q}$  production cross section for nuclear targets by substituting  $\sigma_{gQ\bar{Q}}^A$  for  $\sigma_{gQ\bar{Q}}^N$  in Eq. (21). In the frozen size approximation the cross section  $\sigma_{gQ\bar{Q}}^A$  for the scattering of a color singlet  $gQ\bar{Q}$  state from a nucleus with mass number A is given by the conventional Glauber formalism:

$$\sigma_{gQ\bar{Q}}^{A} = 2 \int d^{2}\vec{b} \left\{ 1 - \left[ 1 - \frac{1}{2A} \sigma_{gQ\bar{Q}}^{N} T(\vec{b}) \right]^{A} \right\}$$

$$\approx 2 \int d^{2}\vec{b} \left\{ 1 - \exp \left[ -\frac{1}{2} \sigma_{gQ\bar{Q}}^{N} T(\vec{b}) \right] \right\}. \tag{29}$$

Here  $\vec{b}$  is the impact parameter of the  $gQ\bar{Q}$ -nucleus scattering process, which must not be confused with the impact parameter of the  $gQ\bar{Q}$ -nucleon inter-

action in Sec. 3.  $T(\vec{b})$  stands for the optical thickness of the nucleus:

$$T(\vec{b}) = \int_{-\infty}^{+\infty} dz \, n_A(\vec{b}, z) \,, \tag{30}$$

with the nuclear density  $n_A(\vec{b},z)$  normalized to  $\int d^3\vec{r}\, n_A(\vec{r}) = A$ . We then obtain:

$$\frac{d\sigma_{Q\bar{Q}}(h,A)}{dx_b} \approx g_b(x_b, \mu^2 = 4m_Q^2) \,\sigma(GA \to Q\bar{Q}\,X; x_t, k_b^2 = 0), \qquad (31)$$

where

$$\begin{split} \sigma(GA \to & Q\bar{Q}\,X; x_t, k_b^2) = 2 \int d^2\vec{r} \int_0^1 dz \, |\Psi_G(z,r)|^2 \\ & \times \int d^2\vec{b} \left\{ 1 - \exp\left[ -\frac{1}{2} \, \sigma^N_{gQ\bar{Q}} T(\vec{b}) \right] \right\} \equiv \left\langle \sigma^A_{gQ\bar{Q}} \right\rangle \,. \end{split} \tag{32}$$

In the multiple scattering series (29, 32) nuclear coherence effects are controlled by the parameter

$$\tau_A = \sigma_{gQ\bar{Q}}^N T(\vec{b}) \propto \sigma_{gQ\bar{Q}}^N A^{1/3}. \tag{33}$$

Expanding the exponential in Eq.(32) in powers of  $\tau_A$  one can identify terms proportional to  $\tau_A^n$  which describe contributions to the total production cross section resulting from the coherent interaction of the  $gQ\bar{Q}$  state with n nucleons inside the target nucleus. In leading order, n=1, we have the incoherent sum over the nucleon production cross sections:

$$\sigma(GA \to Q\bar{Q}X) = \int d^2\vec{b} T(\vec{b}) \, \sigma(GN \to Q\bar{Q}X) 
= A \, \sigma(GN \to Q\bar{Q}X). \tag{34}$$

This is the conventional impulse approximation component of the nuclear production cross section, proportional to the nuclear mass number. Effects of coherent higher order interactions are usually discussed in terms of the nuclear transparency

$$T_A = \frac{\sigma_{Q\bar{Q}}(h, A)}{A \,\sigma_{Q\bar{Q}}(h, N)} \,. \tag{35}$$

In the impulse approximation  $T_A=1$ . The driving contribution to nuclear attenuation results from the coherent interaction of the three-parton  $gQ\bar{Q}$  state with two target nucleons:

$$T_A = 1 - \frac{1}{4} \frac{\left\langle \left( \sigma_{gQ\bar{Q}}^N \right)^2 \right\rangle}{\left\langle \sigma_{gQ\bar{Q}}^N \right\rangle} \int d^2\vec{b} \, T^2(\vec{b}) \,. \tag{36}$$

Using Gaussian nuclear densities fitted to the measured electromagnetic charge radii,  $R_{ch} \approx r_0 A^{1/3} \approx 1.1 \, fm \, A^{1/3}$ , we obtain:

$$T_A = 1 - \frac{3}{16\pi r_0^2} \frac{\left\langle \left(\sigma_{gQ\bar{Q}}^N\right)^2\right\rangle}{\left\langle \sigma_{gQ\bar{Q}}^N\right\rangle} A^{1/3}. \tag{37}$$

In Fig.3 we present results for the transparency ratio  $T_A$  using the two different parameterizations for the dipole cross section, introduced in the previous section (see Eqs.(19,28)). The dipole cross section from Eq.(28) depends in general on  $x_t$ , which is however restricted by the kinematic constraint  $x_t x_b s \geq 4m_c^2$ . We choose  $x_t = 0.01$  — a typical value for current Tevatron energies ( $s \sim 1600\,GeV^2$ ). For both parameterizations of the dipole cross section we find only small nuclear effects. In the commonly used parameterization  $T_A = A^{\alpha-1}$  they translate to  $\alpha \sim 0.99$ . This is in agreement with recent experiments at CERN and Fermilab: In the WA82 experiment [17] with a  $340\,GeV$   $\pi^-$  beam one finds  $\alpha = 0.92 \pm 0.06$ , at an average value of  $\bar{x}_F = 0.24$ . From the E769 measurement [18], using a  $250\,GeV$   $\pi^\pm$  beam,  $\alpha = 1.00 \pm 0.05$  was obtained for  $0 < x_F < 0.5$ .

### 5 Summary

We have presented a light-cone wave function formulation of heavy-flavor production in high energy hadron-hadron collisions. At moderate values of  $x_F$  we have found that heavy-flavor production can be viewed as the diffractive excitation of heavy  $Q\bar{Q}$  Fock states present in the wave function of the interacting projectile gluon. In the region of applicability  $(0 < x_F < 0.5)$  the energy dependence of recent data on open-charm production in proton-proton collisions is described reasonably well. The light-cone wave function formulation is most appropriate to address the production of heavy-flavors from nuclear targets. In accordance with recent experiments on open-charm production we have found small nuclear effects. High precision measurements involving heavy nuclear targets would be necessary to observe any significant nuclear modification of heavy-flavor production rates.

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## Figures:

- Figure 1: Coupling of the exchanged t-channel gluon to different Fock components of the incident gluon from the hadron target. With respect to the gluon-gluon subprocess (a) and (b) correspond to a t- and u-channel quark exchange, and (c) to a s-channel gluon exchange process.
- Figure 2: The differential cross section  $d\sigma_{c\bar{c}}/dx_F$  for open charm production in nucleon-nucleon collisions at  $E_{lab} = 800 \, GeV$  calculated via Eq.(27). For the full (dashed) curve the dipole cross section from Eq.(19) (Eq.(28)) was used. The experimental data are from ref.[14].
- Figure 3: The A dependence of the nuclear transparency  $T_A$ . For the full curve the dipole cross section from Eq.(19) was used. The dashed curve was obtained with the dipole cross section from Eq.(28) for  $x_t = 0.01$ .









